Blind Sensor Calibration in Sparse Recovery

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***Abstract*—We consider the problem of calibrating a compressed sensing measurement system under the assumption that the decalibration consists of unknown complex gains on each measure. We focus on *blind* calibration, using measures performed on a few unknown (but sparse) signals. In the considered context, we study several sub-problems and show that they can be formulated as convex optimization problems, which can be solved easily using off-the-shelf algorithms. Numerical simulations demonstrate the effectiveness of the approach even for highly uncalibrated measures, when a sufficient number of (unknown, but sparse) calibrating signals is provided.**

I. Introduction

We consider the blind calibration problem in a system with sensors having effective unknown complex valued gain and a number of unknown sparse training signals, $\mathbf{x}_l \in \mathbb{C}^N, \quad l = 1 \dots L$. The measured signal, $y_{i,l} \in \mathbb{C}$, in this system is modeled as

$$y_{i,l} = d_i e^{j\theta_i} \mathbf{m}_i^* \mathbf{x}_l \qquad i = 1 \dots M \qquad .^* : \text{Conj. Transpose} \tag{1}$$

where $\mathbf{m}_i \in \mathbb{C}^N$ are known sensor projection vectors, $d_i \in \mathbb{R}^+$ are unknown gain magnitude and $\theta_i \in [-\pi, \pi]$ are the unknown phase shifts for each sensor. This problem can be simplified to 2 sub-problems for easier analysis.

II. Gain Calibration

For known phases, the calibration problem can be formulated as a convex optimization problem such that

$$\mathbf{x}_{l,*}, \Delta_{i,*} = \arg\min_{\substack{\mathbf{x}_1,\dots,\mathbf{x}_L \\ \Delta_1,\dots,\Delta_M}} \quad \sum_{l=1}^{L} \|\mathbf{x}_l\|_1 \tag{2}$$

$$\text{subject to} \quad \sum_{i=1}^{M} \Delta_i = c, \quad \Delta_i y_{i,l} = \mathbf{m}_i^* \mathbf{x}_l, \quad \begin{matrix} l = 1 \dots L \\ i = 1 \dots M \end{matrix}$$

where $c > 0$ is an arbitrary constant and the resulting estimated gains are $d_{i,*} = 1/\Delta_{i,*}$. This optimization problem has been investigated in [1]. The presented results show that, if there are sufficient number of training signals, the calibration approach provides significantly better performance than traditional recovery (by solving (2) with $\delta_i = 1, i = 1, \dots, M$) when the gain magnitudes have high variance.

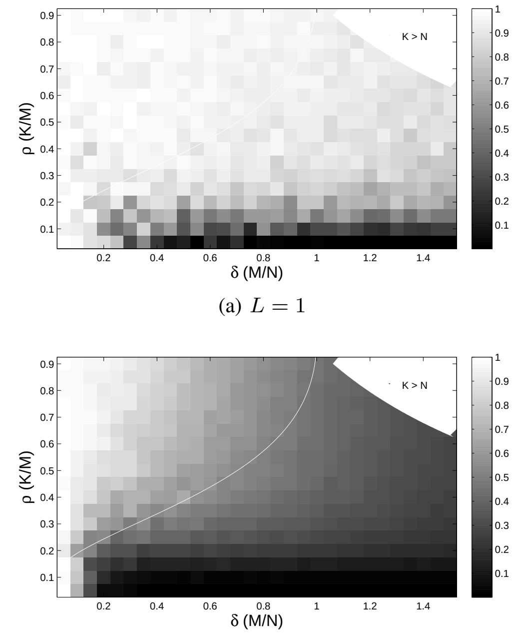
III. Phase Calibration

For known gains, the calibration problem is reduced to estimating the unknown phases. In case of single sparse training signal, this problem is equivalent to the phase retrieval problem investigated in [2]. When dealing with multiple sparse signals ($\mathbf{x}_l \in \Sigma_K$), we propose to perform the calibration and signal estimation with the semidefinite programming

$$\mathbf{X}_* = \arg\min_{\mathbf{X}} \quad \mathrm{Tr}(\mathbf{X}) + \lambda \|\mathbf{X}\|_1 \tag{3}$$

$$\text{subject to} \quad \mathbf{X} \succcurlyeq 0, \quad y_{i,k} y_{i,l}^* = \mathbf{m}_i^* \mathbf{X}_{k,l} \mathbf{m}_i, \quad \begin{matrix} k, l = 1 \dots L \\ i = 1 \dots M \end{matrix}$$

$$\mathbf{X}_{k,l} \triangleq \mathbf{x}_k \mathbf{x}_l^* \in \mathbb{C}^{N \times N}, \quad \mathbf{X} \triangleq \begin{bmatrix} \mathbf{X}_{1,1} & \cdots & \mathbf{X}_{1,L} \\ \vdots & & \vdots \\ \mathbf{X}_{L,1} & \cdots & \mathbf{X}_{L,L} \end{bmatrix} \in \mathbb{C}^{LN \times LN}$$



(a) $L = 1$

(b) $L = 10$

Fig. 1: The decorrelation, σ_I, between the source signal and the estimated signal averaged over 10 randomly generated simulations for various ρ and δ ($\sigma_I(\mathbf{x}_1, \mathbf{x}_2) \triangleq 1 - \frac{|\mathbf{x}_1^* \mathbf{x}_2|^2}{\|\mathbf{x}_1\|_2^2 \|\mathbf{x}_2\|_2^2}$). The Donoho-Tanner phase transition curve is indicated with the white line.

which minimizes the rank and sparsity of the joint signal matrix $\mathbf{X}$. The resulting estimated signal, $\mathbf{x}_* = [\mathbf{x}_1^* \cdots \mathbf{x}_L^*]^*$ is the eigenvector of $\mathbf{X}$ that corresponds to the largest eigenvalue and the estimated phase shifts, $\theta_{i,*}$, are easily computed given $\mathbf{x}_*$ and $y_{i,l}$. Sample simulation results comparing the joint optimization in (3) to the independent optimization described in [2] can be seen in Figure 1, which shows much higher correlation with the reconstructed signal for $L = 10$.

The talk will present further performance analysis of the proposed method for phase calibration, and discuss methods combining the gain and phase calibration approaches for calibration of complex valued gains.

References

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[2] H. Ohlsson, A. Y. Yang, R. Dong, and S. S. Sastry, "Compressive Phase Retrieval From Squared Output Measurements Via Semidefinite Programming", Accepted to the 16th IFAC Symposium on System Identification, SYSID 2012.